

Resonance absolute quantum reflection at selected energies

V. M. Chabanov and B. N. Zakhariev

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Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research,
141980 Dubna, Russia
email: zakharev@thsun1.jinr.ru

Abstract

The possibility of the resonance reflection (100 % at maximum) is revealed. The corresponding exactly solvable models with the controllable numbers of resonances, their positions and widths are presented.

The foundation of quantum mechanics is, after all, the laws of wave motion in different potentials. The most interesting are the potentials with the especial qualitative properties because they give a deeper insight into the peculiarities of the microworld. We have revealed the phenomenon of *total reflection* at selected energy points by specific potentials. It is remarkable that this occurs even for *above-barrier* motion. The possibility itself of such a resonance reflection has surprisingly never been mentioned since the very beginning of the wave mechanics.

Let us consider the one-dimensional Schrödinger equation on the whole axis $(-\infty < x < \infty)$ with time-independent potentials. For our purpose, we shall need the potentials of Neumann-Wigner type which have bound states embedded into continuum (BSEC) on the *semi-axis* $0 \leq x < \infty$ [1, 2, 3, 4]. The expression for the potential in the simplest case of the only bound state is the following

$$V(x) = -2 \frac{d}{dx} \left\{ \frac{c^2 \sin(kx)^2}{k^2 \left[1 + \frac{c^2}{k^2} \left(\frac{x}{2} - \frac{\sin(2kx)}{4k} \right) \right]} \right\},$$

$$k = \sqrt{E_{BSEC}}. \quad (1)$$

The BSEC wave function normalized to unity at the positive energy value $E = E_{BSEC} > 0$ has the form

$$\Psi(x, k) = \frac{c \sin(kx)}{k \left[1 + \frac{c^2}{k^2} \left(\frac{x}{2} - \frac{\sin(2kx)}{4k} \right) \right]}, \quad (2)$$

see the right-hand-side of Fig.1 for $x \geq 0$. The parameter c is the derivative of the BSEC function (2) at the origin. Both the functions, $V(x)$ (1) and $\Psi(x, k)$ (2),

decrease asymptotically $\sim \frac{1}{x}$, as $x \rightarrow \infty$. Every antinode of the BSEC corresponds to well-barrier block of the potential confining the wave from the right [5]. Besides BSEC, there is another linear independent solution at $E = E_{BSEC}$ which diverges asymptotically as $x \rightarrow \infty$. This solution is unphysical and must be omitted.

If continued to the *whole axis* with $V(x < 0) \equiv 0$, these potentials (1) lose their confinement property because the waves now leak out to the left semi-axis where they move freely. Instead this potentials acquire the new remarkable feature : they give a total reflection at the BSEC energies for the waves incident from the left.

This can be explained in the following way. The BSEC-potentials confine wave on the semi-axis $0 \leq x < \infty$ at the chosen energy $E = E_{BSEC}$. They forbid the wave propagation to the right. So, it is natural to expect complete reflection of the waves incident on these potentials from the left. To be more precise, the only physically acceptable solution on the whole axis must coincide with the BSEC (2) on the semi-axis $x \geq 0$ which asymptotically decreases. Its smooth continuation to the negative semi-axis is a free wave $c \sin(kx)/k$ as shown in Fig.1. This sine is a combination of incident and outgoing waves: $c[\exp(ikx) - \exp(-ikx)]/2ik$. The normalized solution has a unit amplitude of incident wave and the reflected wave with the reflection coefficient $|R(E)_{BSEC}| = 1$.

At the energies differing from $E = E_{BSEC}$ (pinned out point) the potentials (1) do not confine the waves on half-axis and cannot be totally reflective. The modulus of reflection coefficient $|R(E)|$ is shown in Fig.2. Pay also attention to the dependence of $|R(E)|$ on the parameter c . A physical sense of this parameter is that it determines the measure of localization of the BSEC near the origin. With increase in c , the BSEC function (2) is concentrated at $x = 0$ and becomes *delta*-function in the limiting case. Large c values correspond to a width of the reflection resonance. On the contrary, for the smaller c the resonance peak occurs narrower. Thus, one can control the width of the resonance peak at $E = E_{BSEC}$ by varying the parameter c . We can also control the number of the resonance points including their positions. Indeed, they are associated with the BSEC energies on the half-axis which can be created by using the inverse problem formalism (exact solutions generalizing Eqs. (1, 2). So we get a new class of potentials corresponding to the exactly solvable models with resonance reflection.

It is worth to mention that tails of BSEC-potentials on $a \leq x \leq \infty$ for any a are responsible for the wave confinement. So, there can be added almost arbitrary potentials at the left side which would not change the position of the reflection resonance, but deform its shape. Particularly, one can expect a tunneling resonance (with 100% penetrability) near the non-penetrability point. But this details will be considered elsewhere.

There is possible the additional control of BSEC. The space-localization of wave accumulation can be shifted by different combinations of abovementioned potential blocks : well-barrier block shifts maximum of the corresponding wave bump to the left ($< -$) and barrier-well pushes one to the right ($- >$) [5]. It is important that wave function knots at $E = E_{BSEC}$ coincide with even knots of the potential. The change of wave derivatives at the knots for each bump and smooth connections of the neighbor wave-bumps result in decreasing (see Fig.1) (or increasing in the case $- >$) in their relative amplitudes. So, for example, the maximum of BSEC's bump amplitudes can be shifted to the right substituting some ($< -$)-blocks by

($- >$)-blocks:

$$(- > \dots - > \max | \text{ampl.} \Psi_{BSEC} | < - \dots < - ..)$$

Of course, the periodic potentials on the half-axis also have the property of total reflection. This occurs at the energy values belonging to the forbidden spectral zones of the same periodic potential continued to the whole axis, which seems to be almost evident. However periodic potentials are not quadratically integrable as potentials with BSEC and have whole bands of total reflection, unlike our case when absolute non-penetrability occurs at the chosen energy points.

One should not confuse the phenomenon with the total reflection of the waves incident at some angle to a plane of demarcation of different optical mediums. In our case, the total reflection is even for incident waves perpendicular to this plane. The same also concerns the Bragg reflection from parallel crystal lattice planes.

The resonance reflection is also possible in multichannel case on the whole axis: e.g., for M coupled Schrödinger equations [6]. Here, the phenomenon appears to be even more diversified. Unlike the one-channel case there is possibility for short range (exponentially decreasing) BSEC potential matrices. There can be different kinds of BSEC states on the half-axis at the fixed energy value. In the case of M degenerated BSEC states there will be resonance 100% reflection for any combination of incident waves in different channels. There can also coexist M - m BSEC and m scattering states at $E = E_{BSEC}$ on the half-axis. Then on the whole axis there will be m linearly independent combinations of incident waves with total reflection at E_{BSEC} and M - m combinations with the comparatively weak reflection. More details will be published elsewhere.

One might be somewhat surprised that the resonance reflection was not known before in spite of long and intensive investigations of quantum scattering. Maybe it was because of the fact that the possibility itself of this phenomenon is due to existence of potentials with BSEC which were better understood in the inverse problem approach. Particularly, striking was the property of wave confinement with positive energy *above the potential barriers*, as in Fig.1. Besides it was necessary to transform BSEC potentials so that they lose the property to keep BSEC, to admit the wave leakage to the left. Only at the expense of this transformation, the potential acquires the new ability of absolute selective reflectivity.

The classes of exact solvable models considered above contribute significantly to our theoretical notions in wave dynamics and interference for quantum design [2, 7, 8], optics, acoustics, radiowave propagation and possible applications.

In conclusion we want to mention an interesting paper [9] about special potentials with reflection resonances, but with $|R(E)| < 1$ at maximum.

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Figure Captions

FIG.1. The only acceptable solution at $E = E_{BSEC} = 1$; $c = 1$ (solid line) and the corresponding potential which is zero at the negative semi-axis and equal to BSEC-potential at the positive semi-axis. Pay attention to the coincidence of BSEC knots with the even knots of BSEC-potential. It is this exact correlation that provides the confinement the waves from the right at the pinned out point E_{BSEC} .

FIG.2. The modulus of the reflection coefficient $|R(E)|$ for potentials having bound states embedded into continuum spectrum (BSEC) on the half-axis $0 \leq x < \infty$ at energy $E_{bound} = 10$ with resonance nonpenetrability at this point for waves on the whole axis. With increasing of BSEC spectral weight parameter C the width of resonance in $|R(E)|$ near the $E_{bound} = 10$ becomes greater. For $C \rightarrow 0$ this width converges to zero: the dashed line corresponds to the limiting peak in $R(E)$.

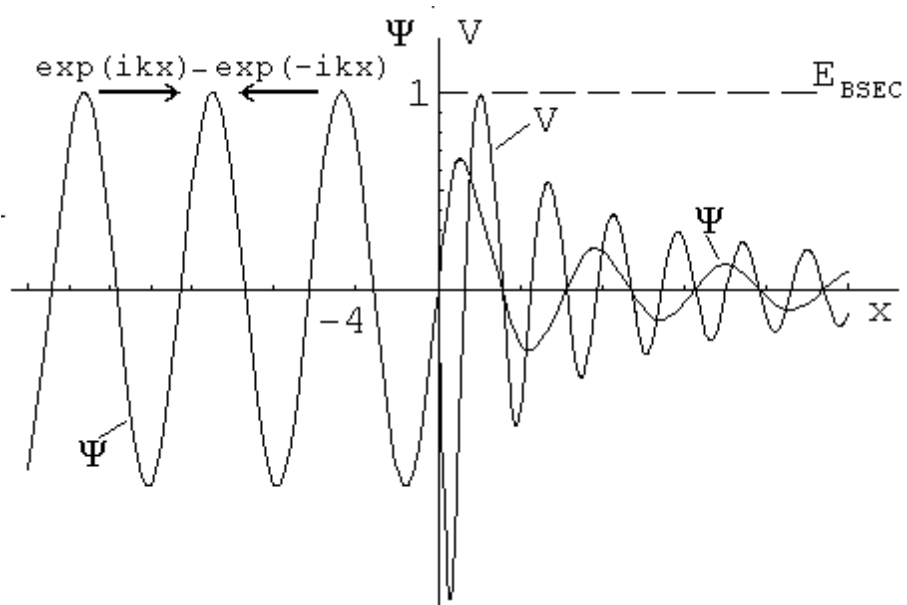


FIG 1. Chabanov et al, PRL 'Resonance Absolute Reflection...'

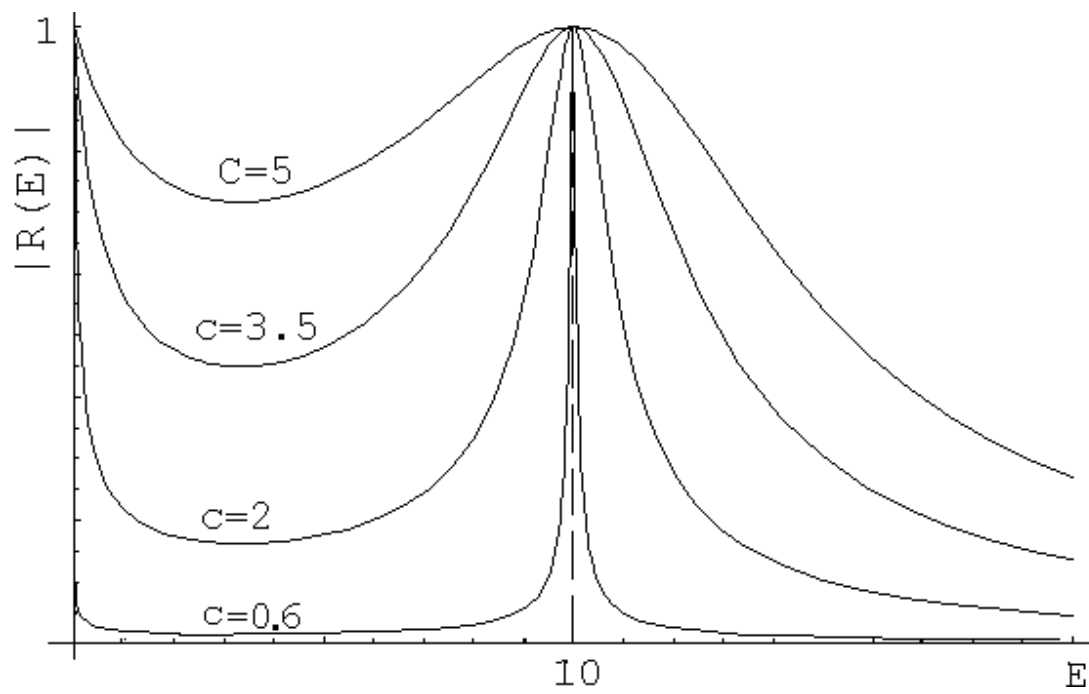


FIG 2. Chabanov et al, PRL 'Resonance Absolute Reflection...'